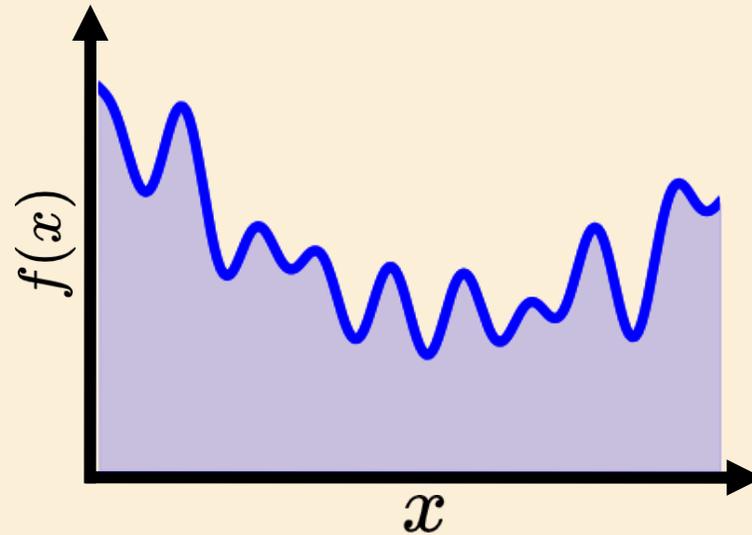


# Adaptive Multiple Control Variates for Many-Light Rendering

Xiaofeng Xu, Lu Wang  
Shandong University, China

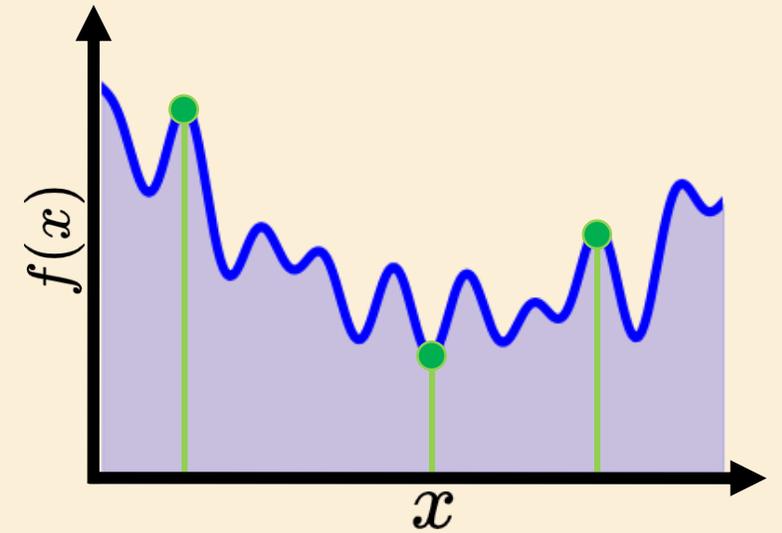
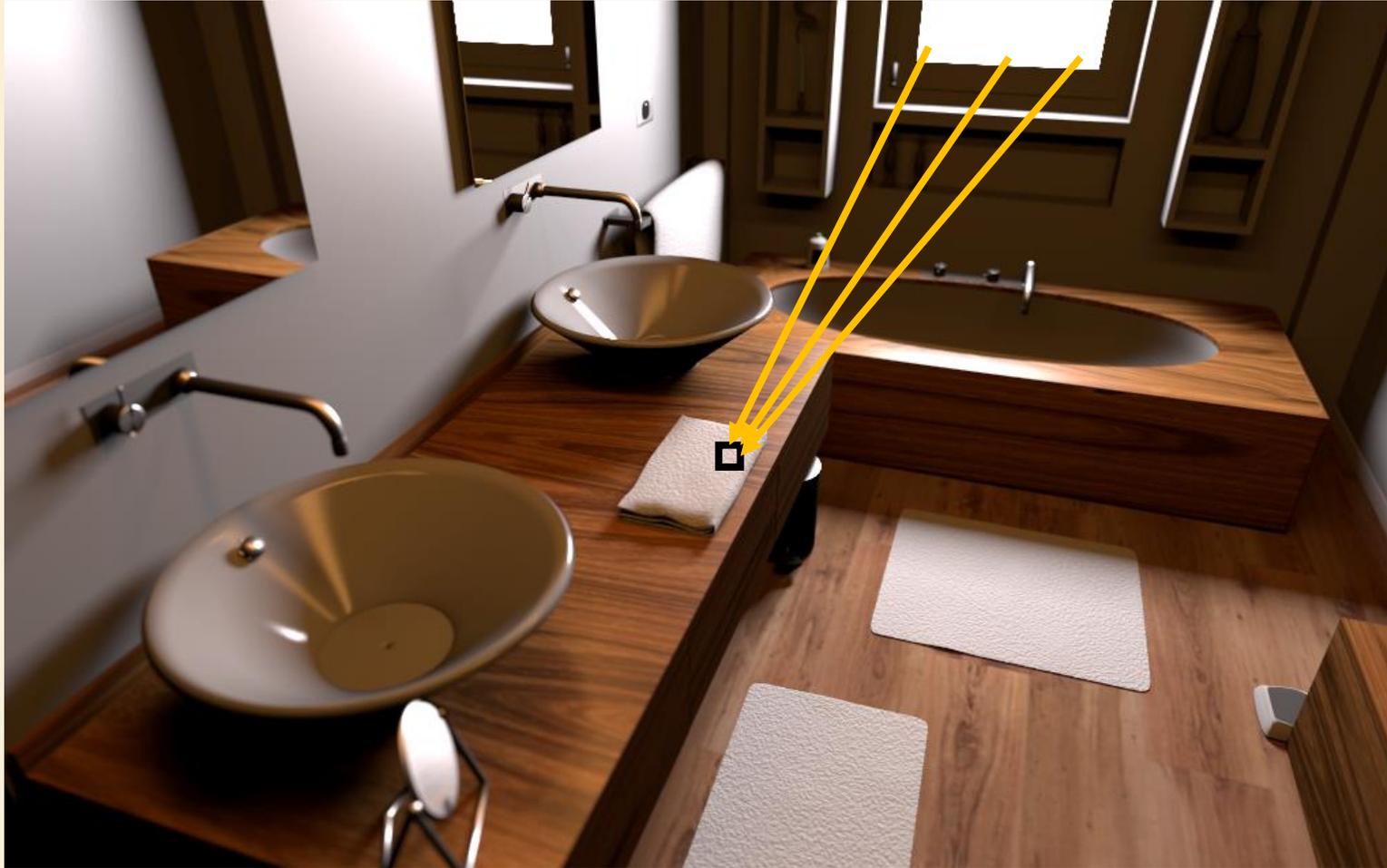


# ■ Problem Statement

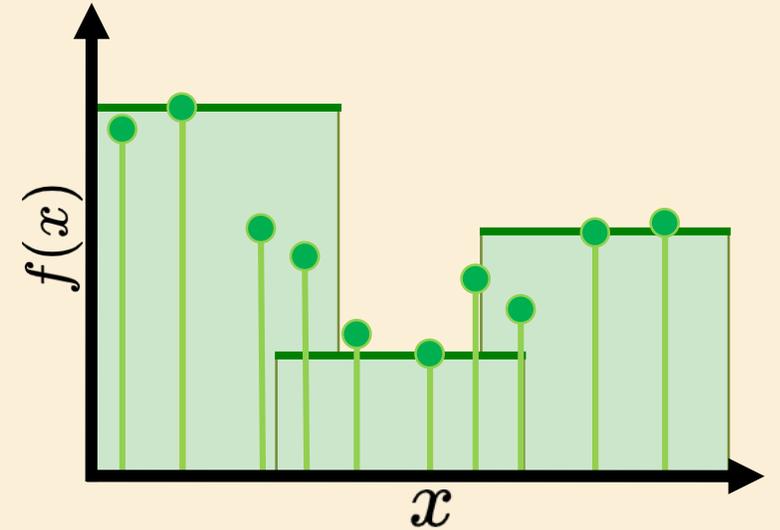
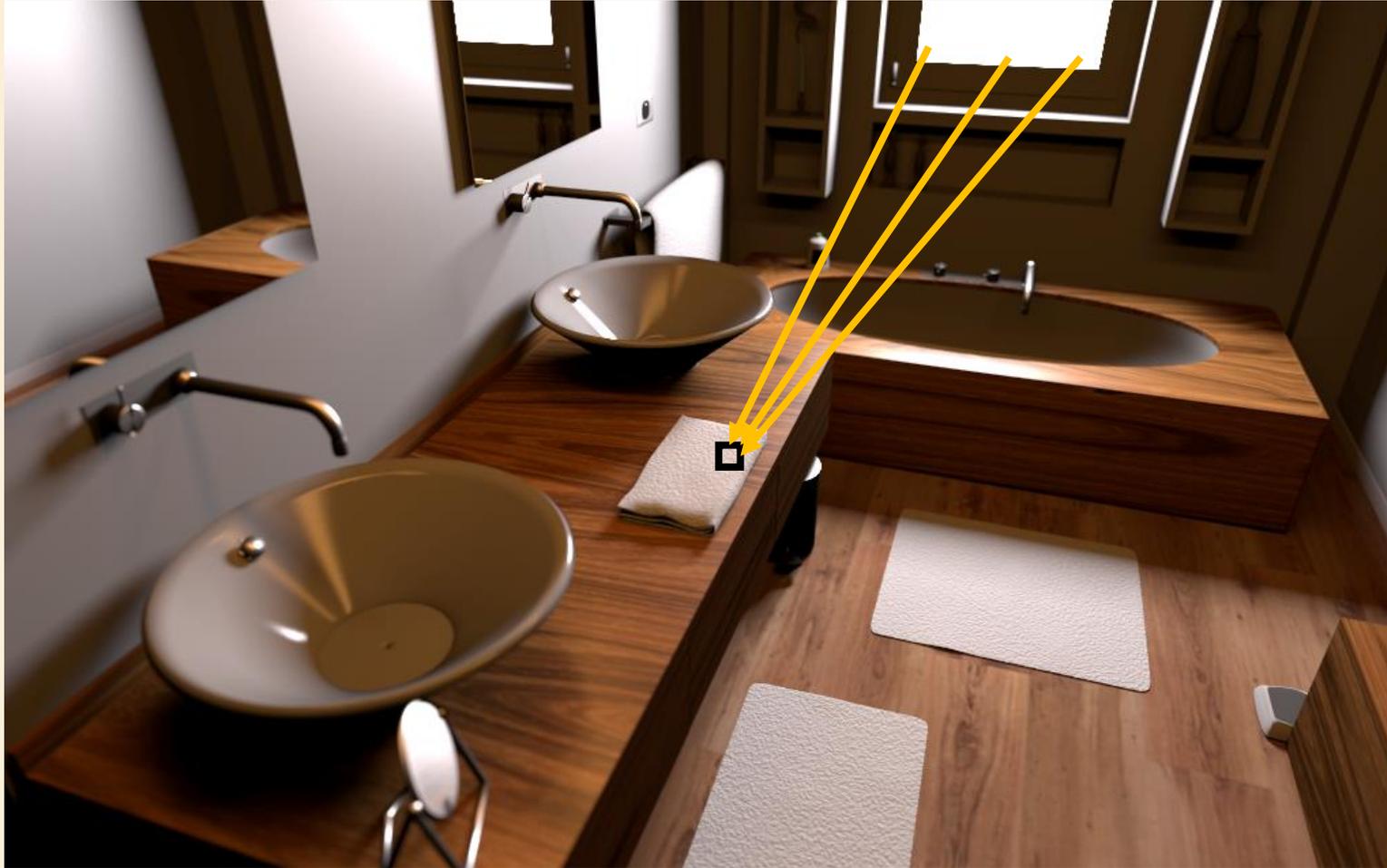


$$F = \int_{[0,1]^D} f(x) dx$$

# Light Transport Context

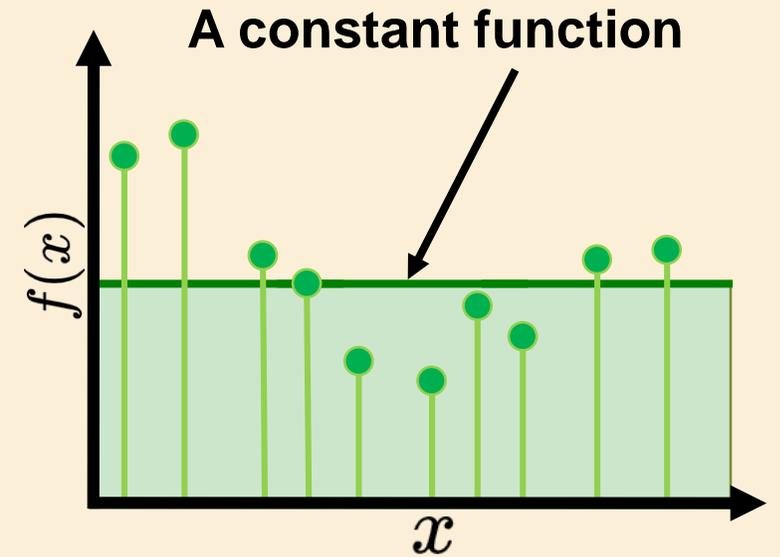


# Monte Carlo



$$\langle F \rangle = \frac{1}{N} \sum_{i=1}^N f(x_i)$$

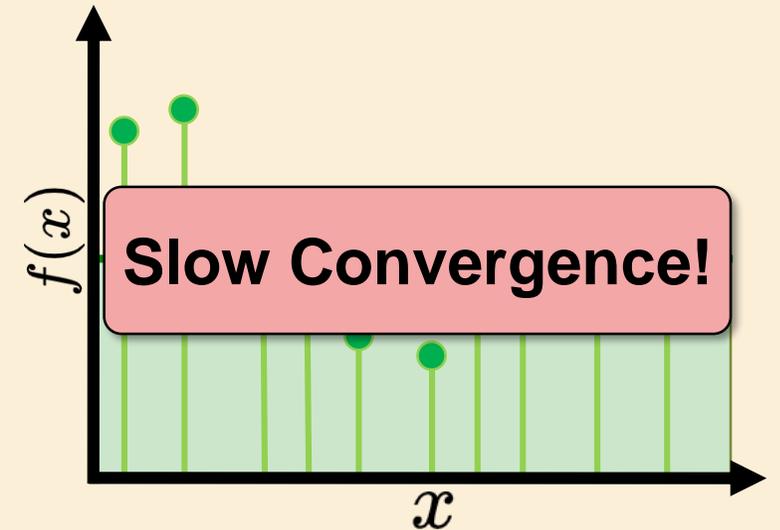
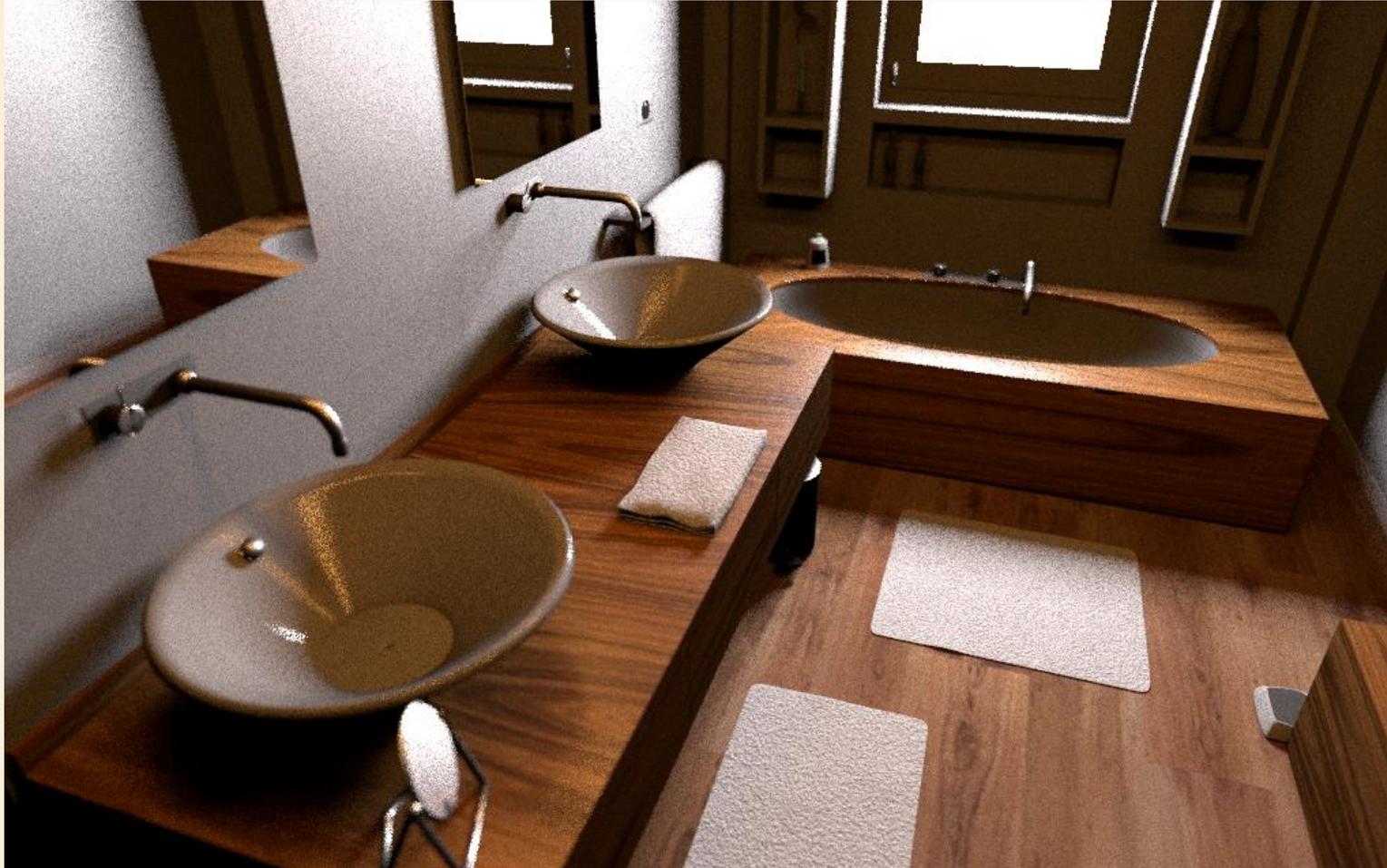
# ■ Calculus-oriented interpretation



$$\langle F \rangle = \frac{1}{N} \sum_{i=1}^N f(x_i)$$

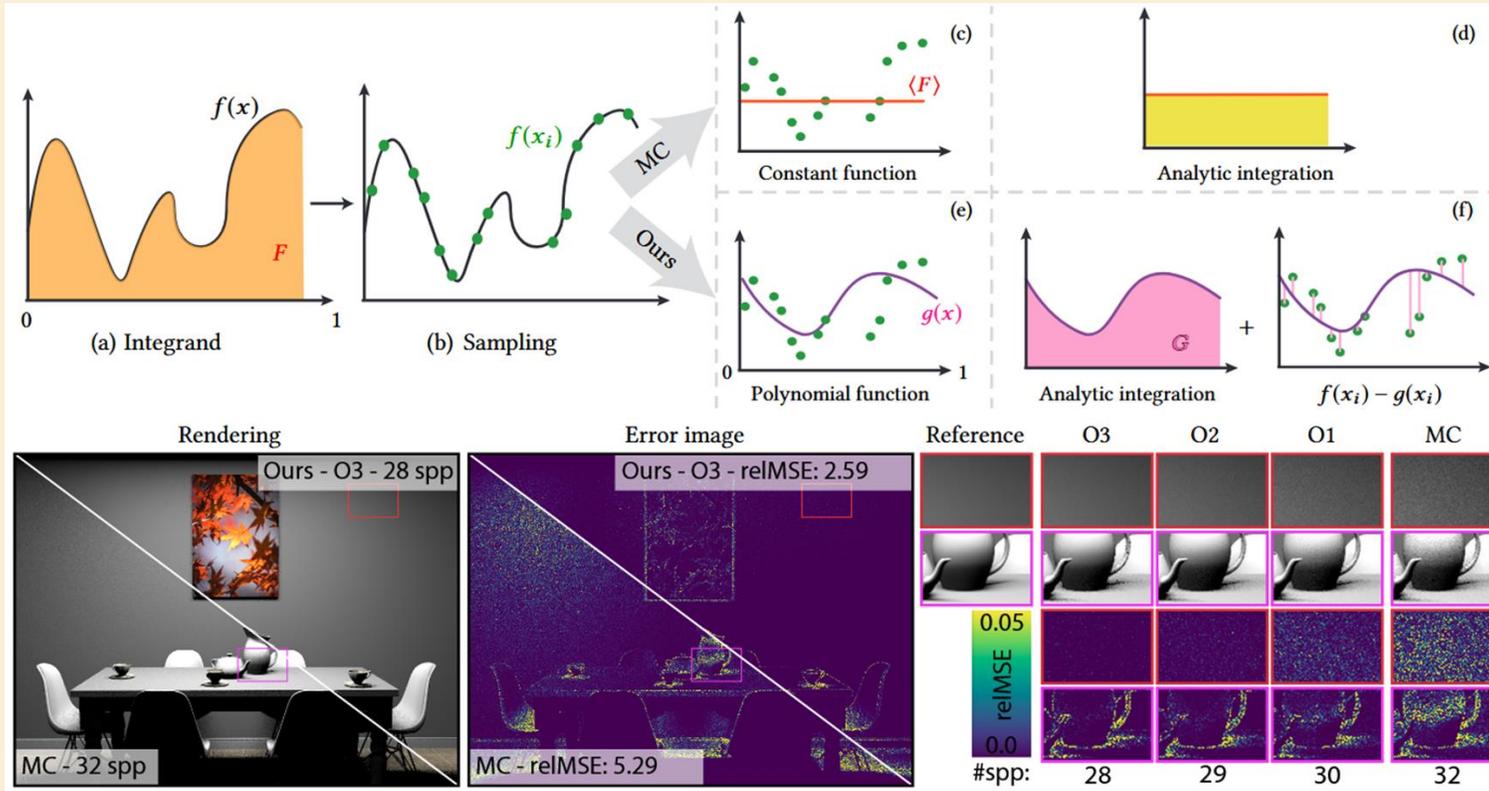
# Limitations of Monte Carlo Integration

256 samples per pixel



$$\langle F \rangle = \frac{1}{N} \sum_{i=1}^N f(x_i)$$

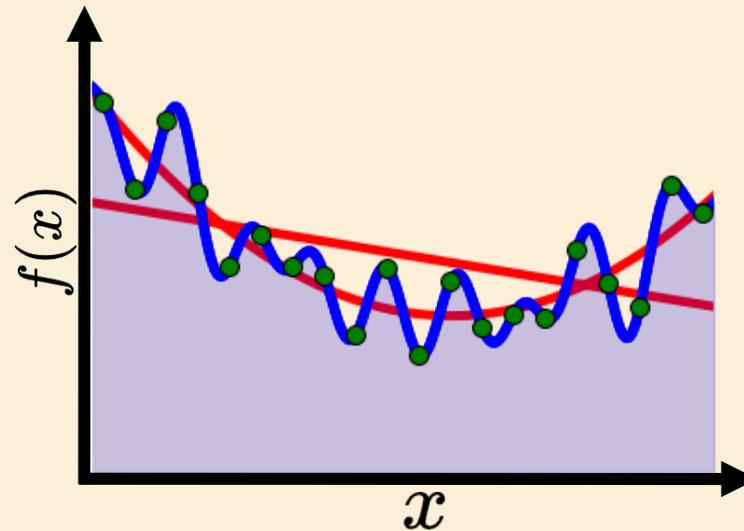
# Regression-based Monte Carlo Integration



Regression

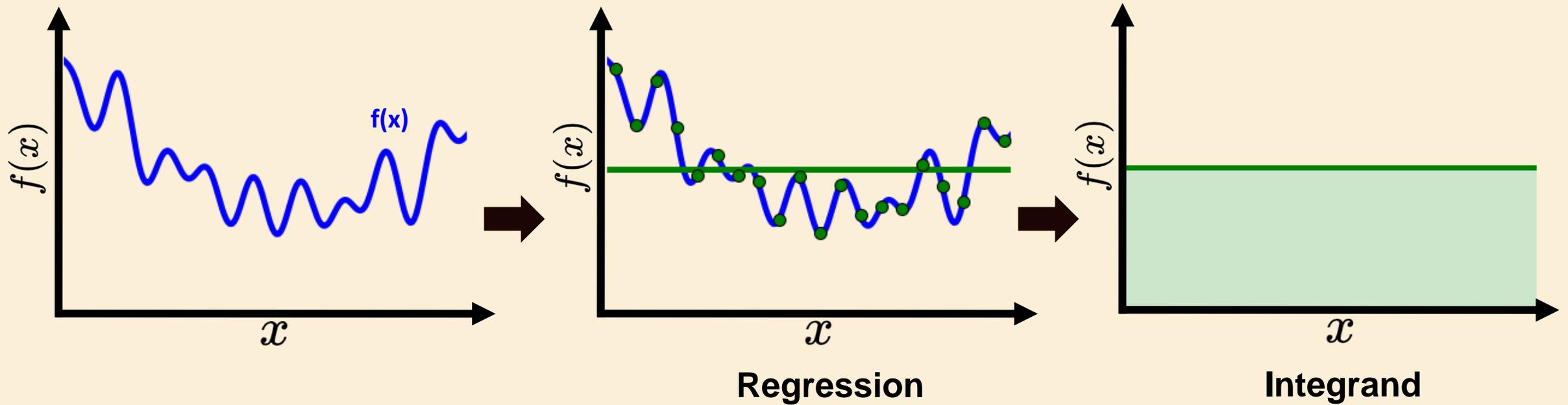
Control Variates

MC Integration

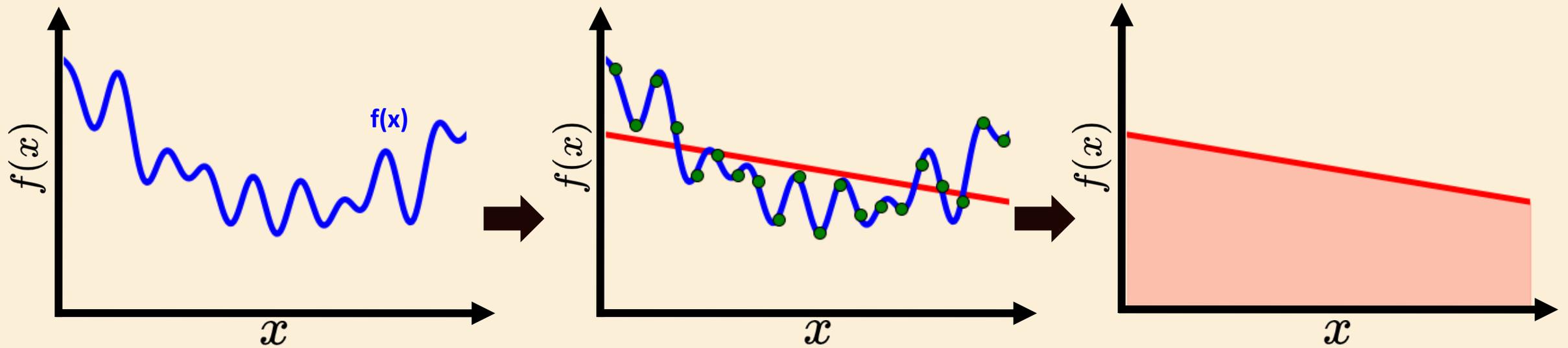


[Salaün et al. 2022] proposed using a more complex function instead of a constant function.

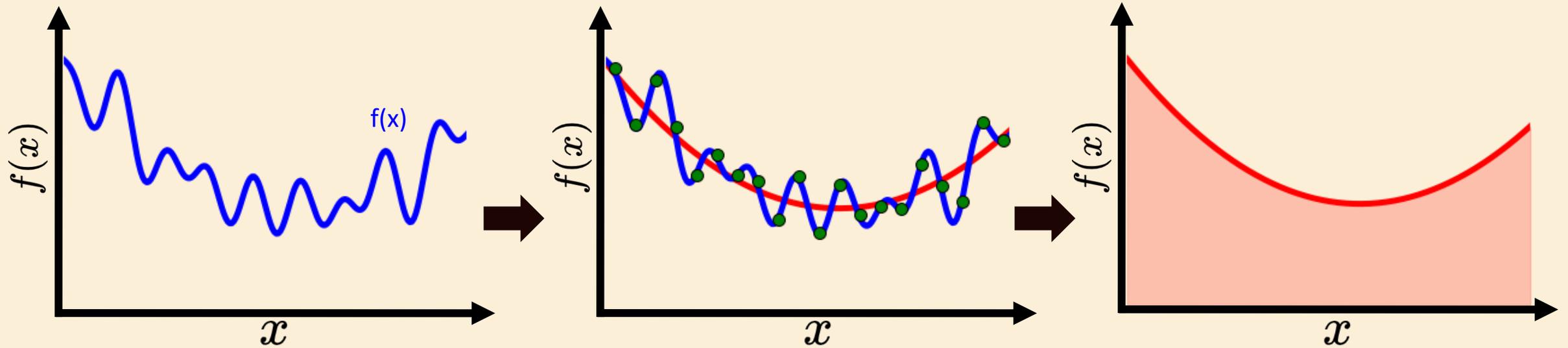
# ■ Regression



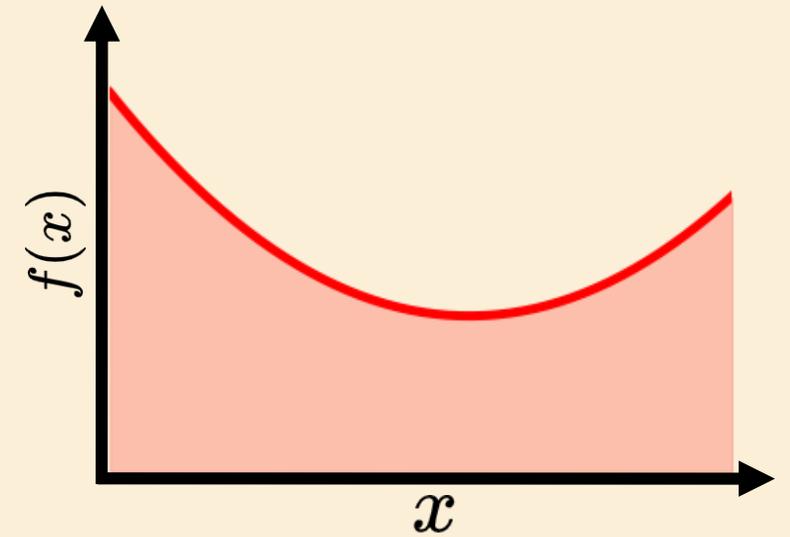
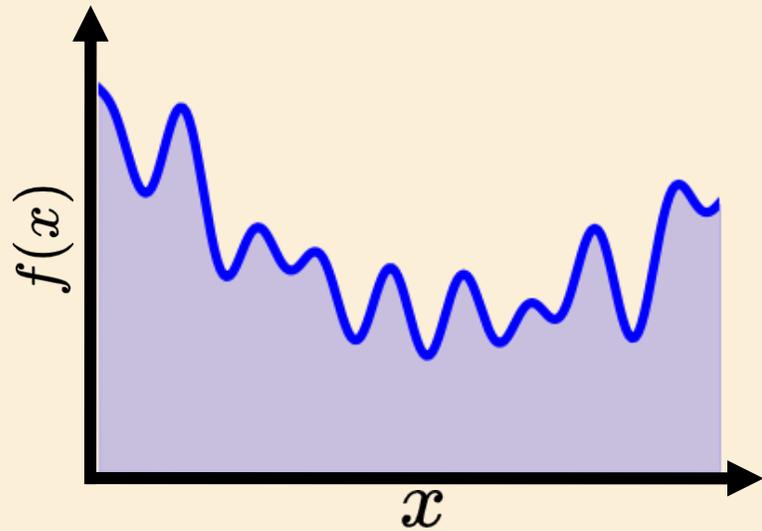
# ■ Regression



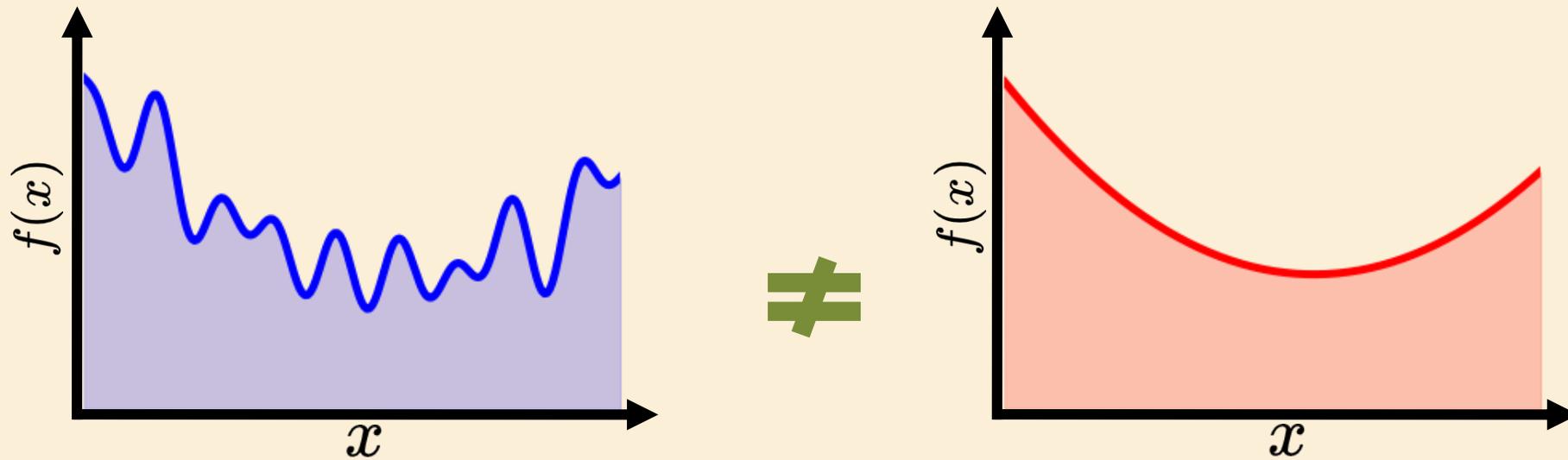
# ■ Regression



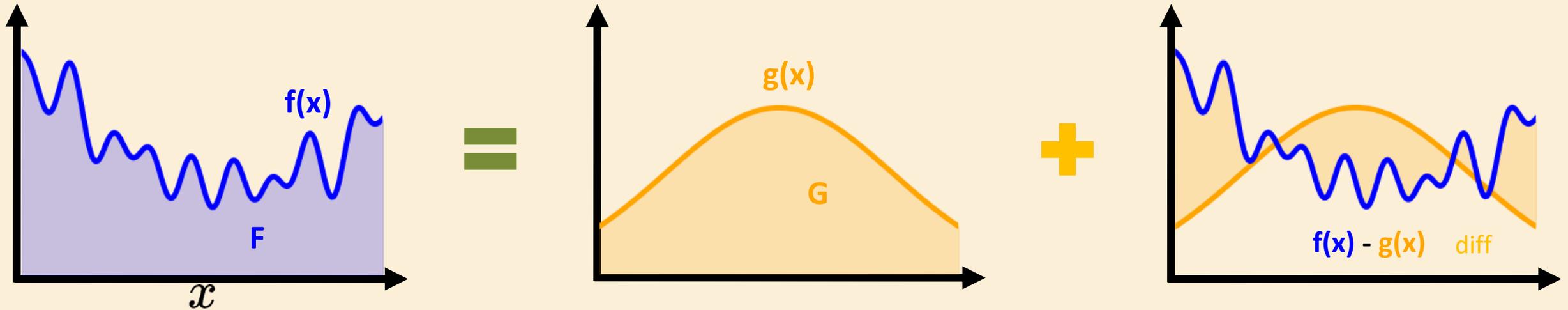
# ■ Regression Leads to Biased Result



# Regression Leads to Biased Result

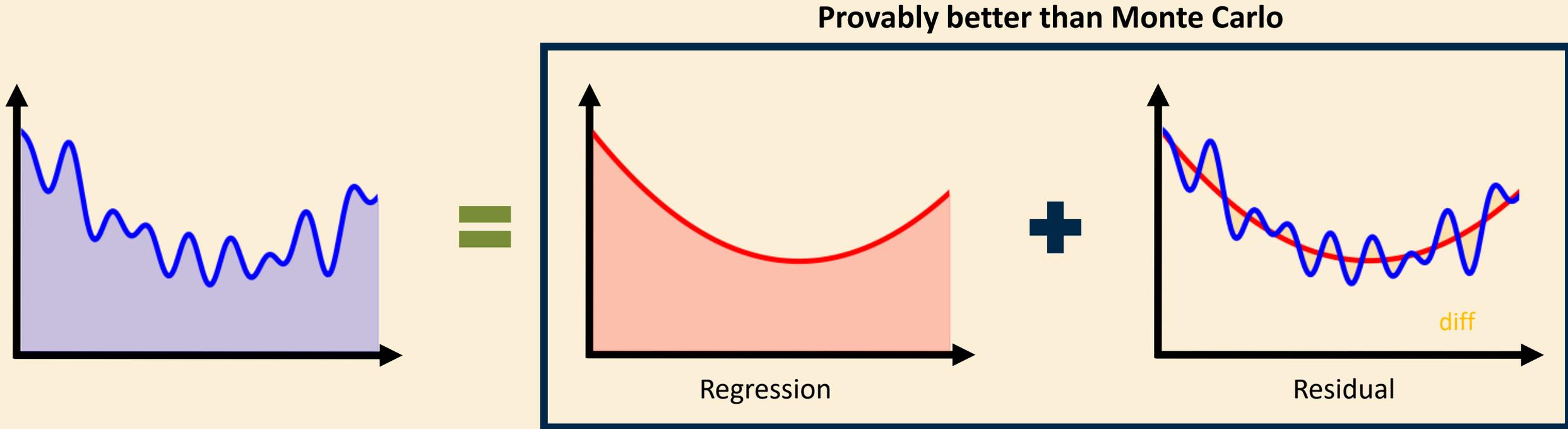


# Control Variates Estimator



$$F = G + (F - G)$$

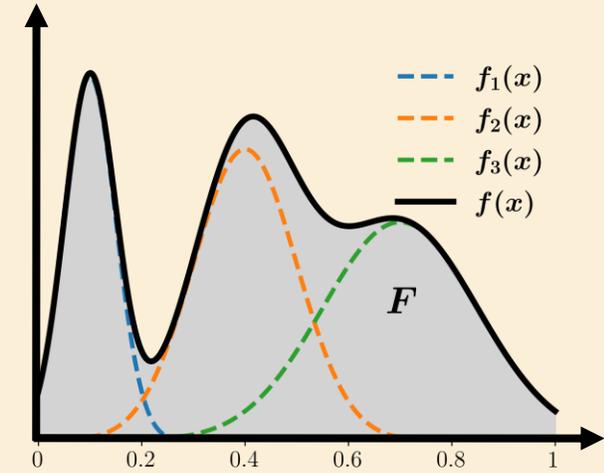
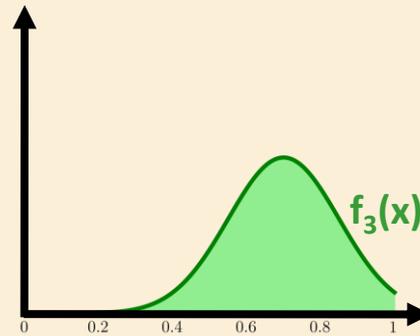
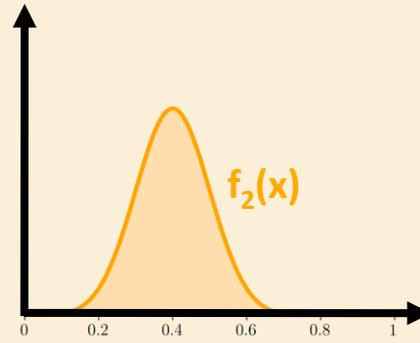
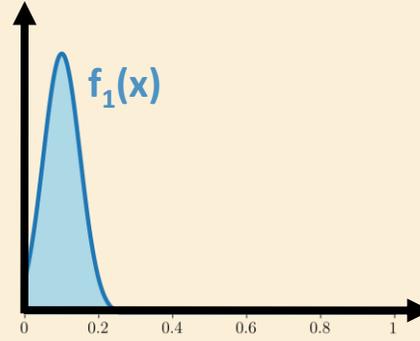
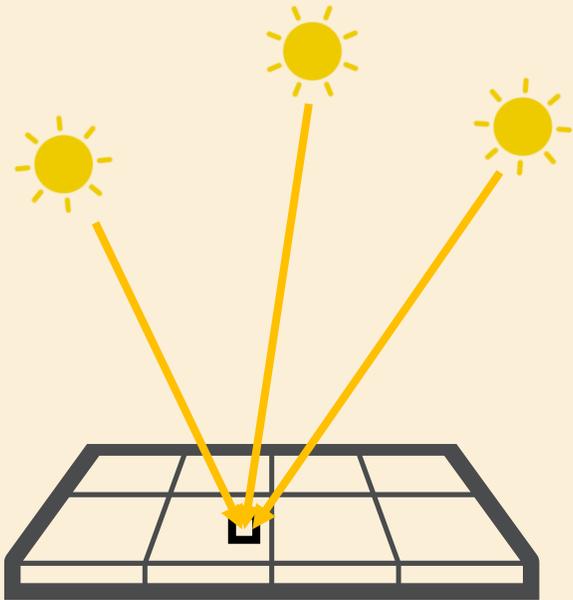
# Regression-based Monte Carlo Integration



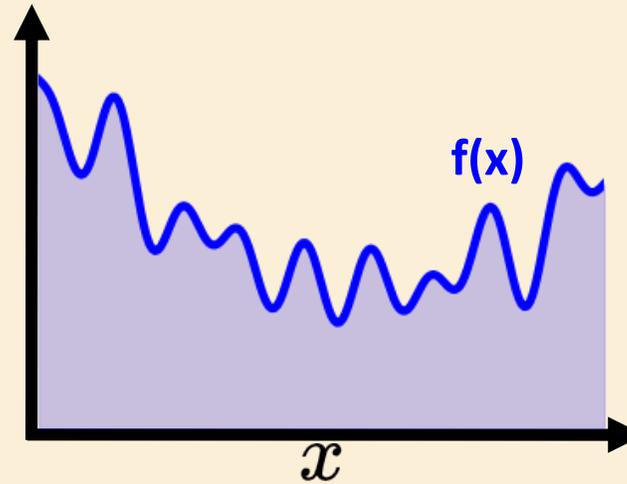
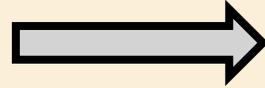
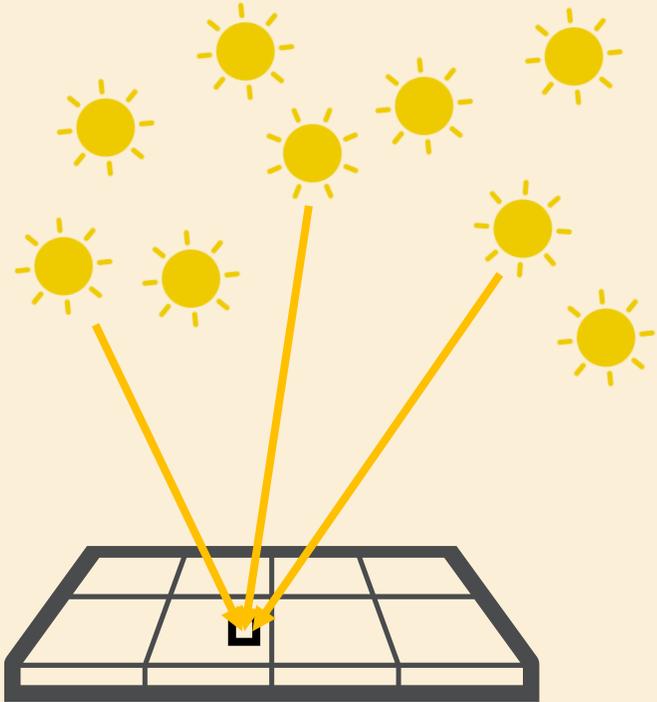
## ■ Challenges

- The least-squares regression solution is not exact for any finite number of samples.
- The integrand can be highly discontinuous due to complex scene configurations (especially in scenes with many lights or complex visibility).

# ■ An Example Scene



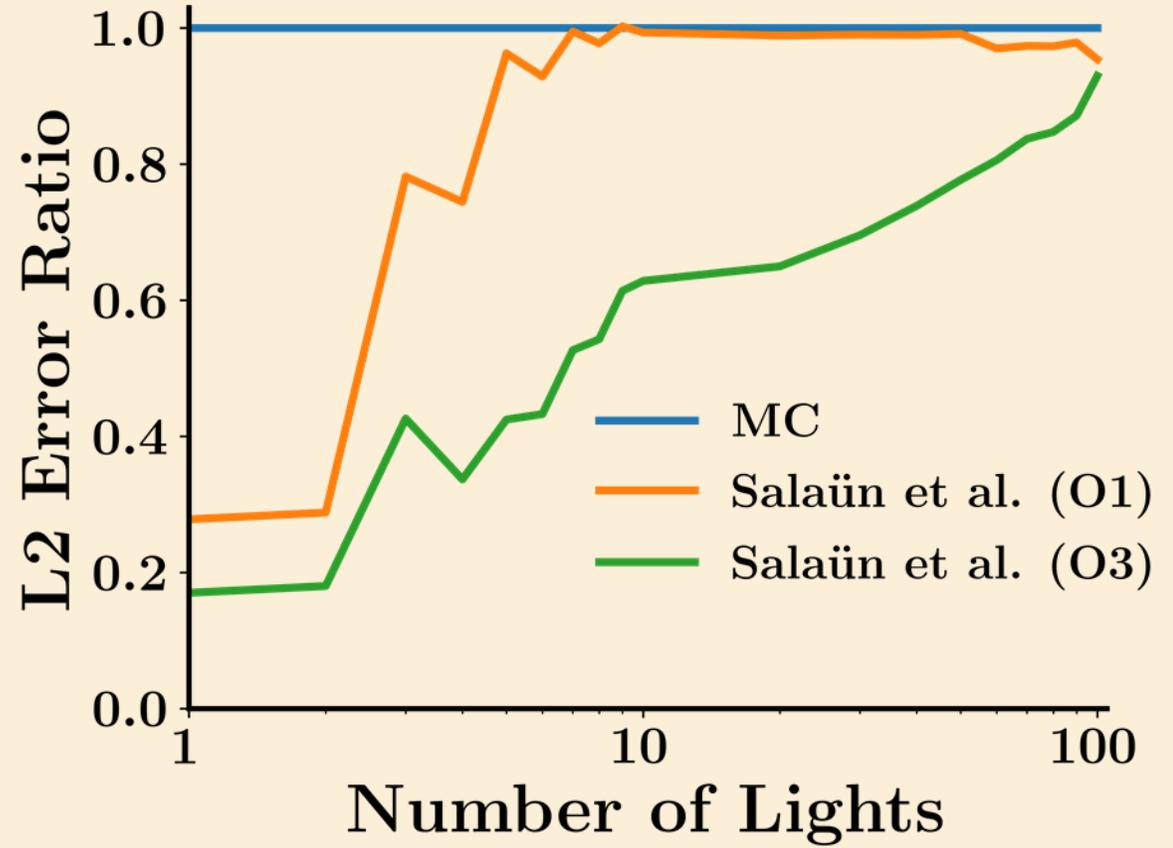
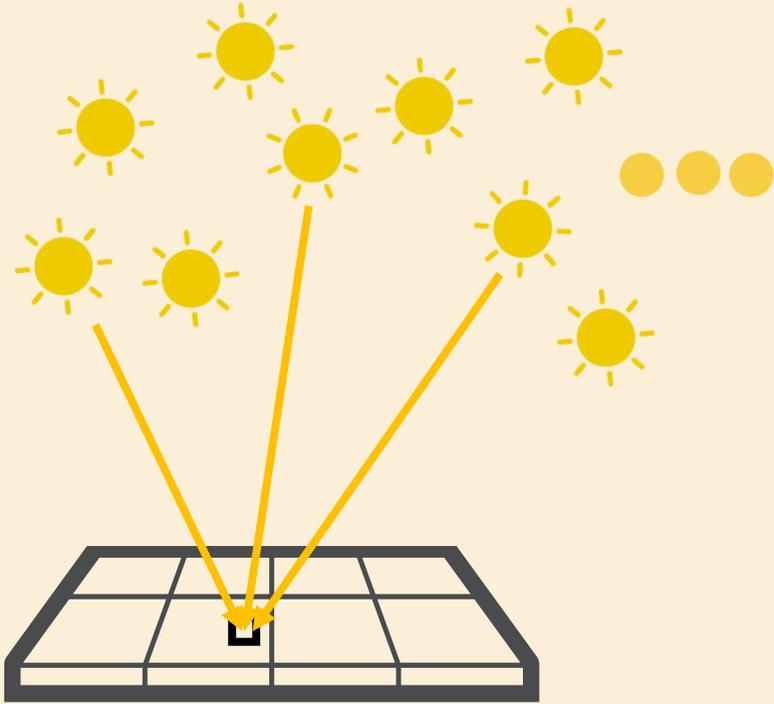
# ■ An Example Scene



**High-frequency!**

**Hard for regression!**

# Many-light Discontinuity



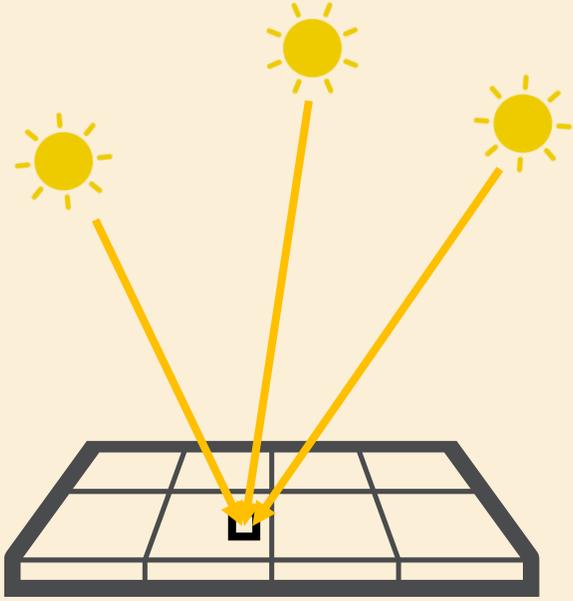
②

①

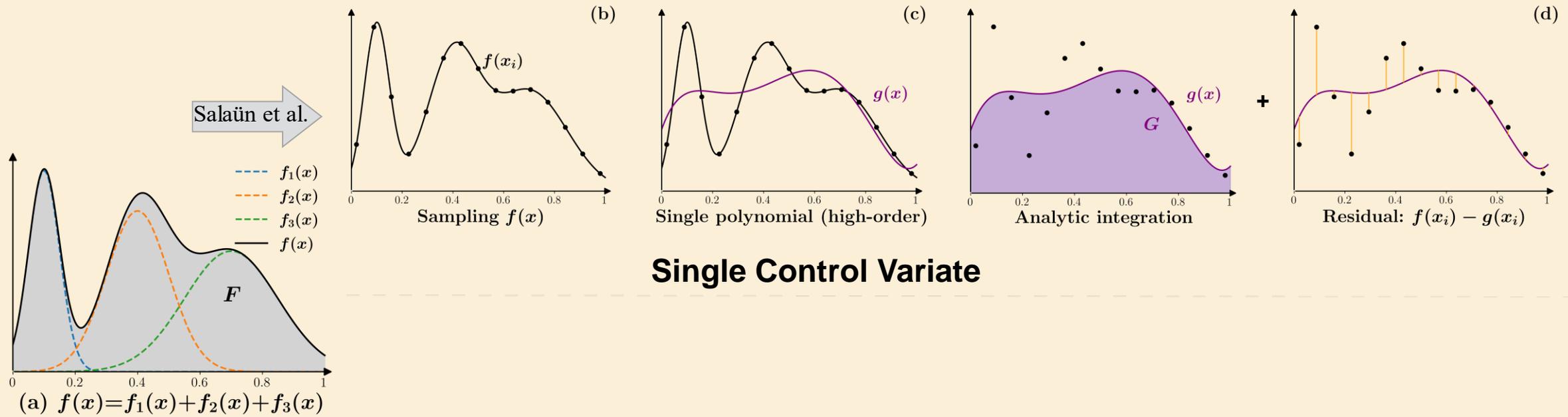
# Adaptive Multiple Control Variates

---

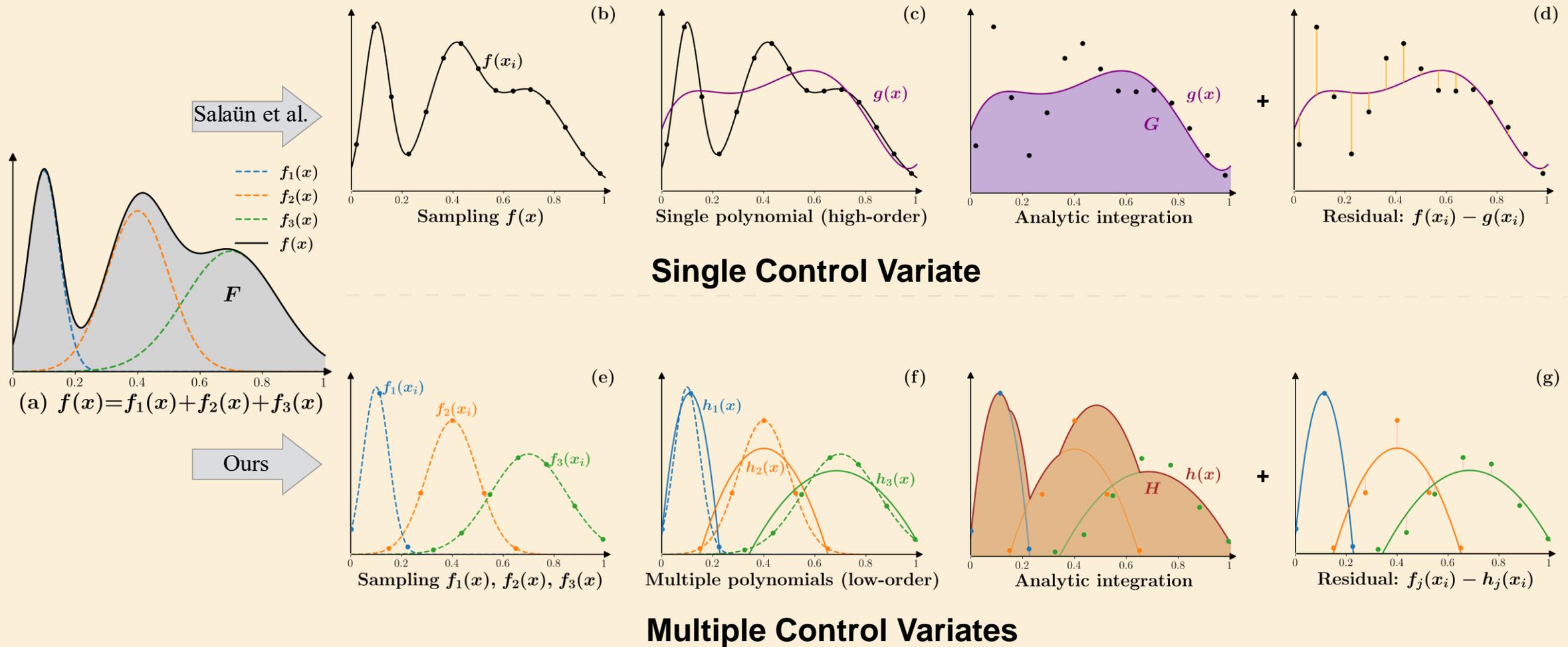
# ■ An Example Scene



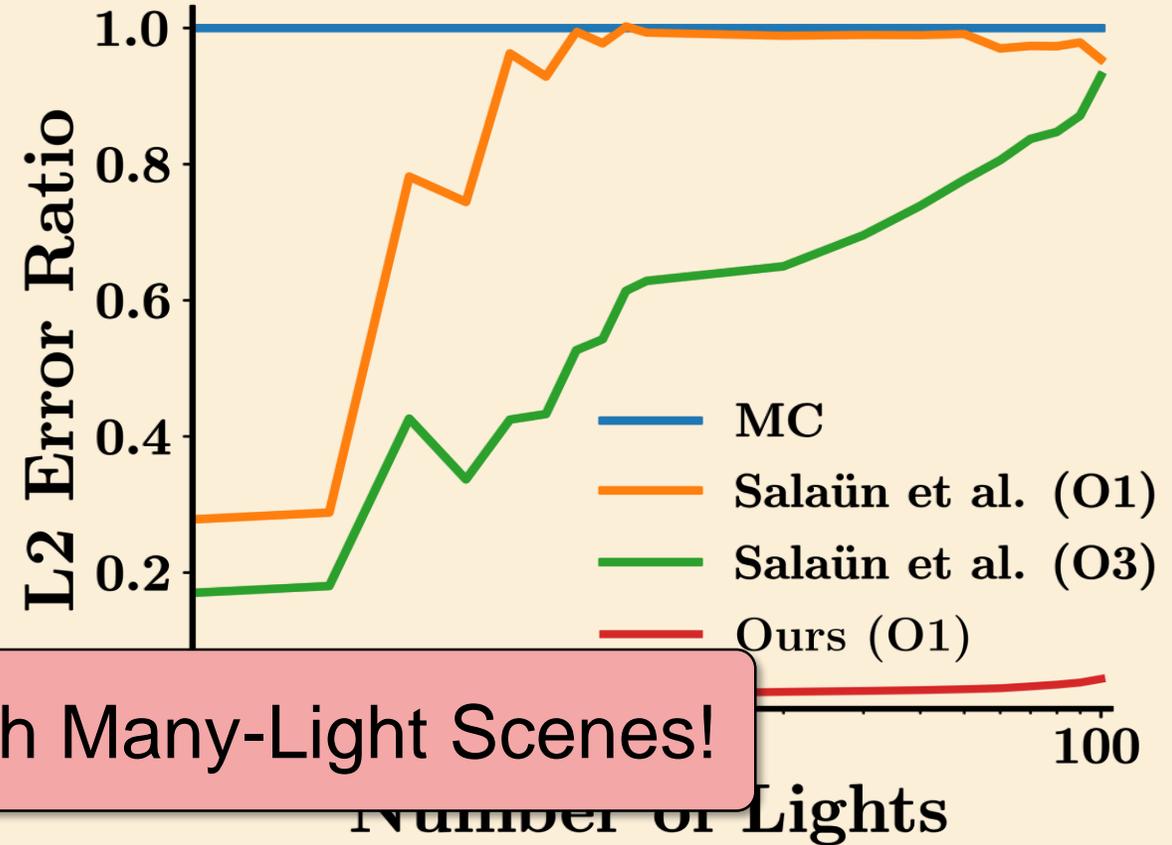
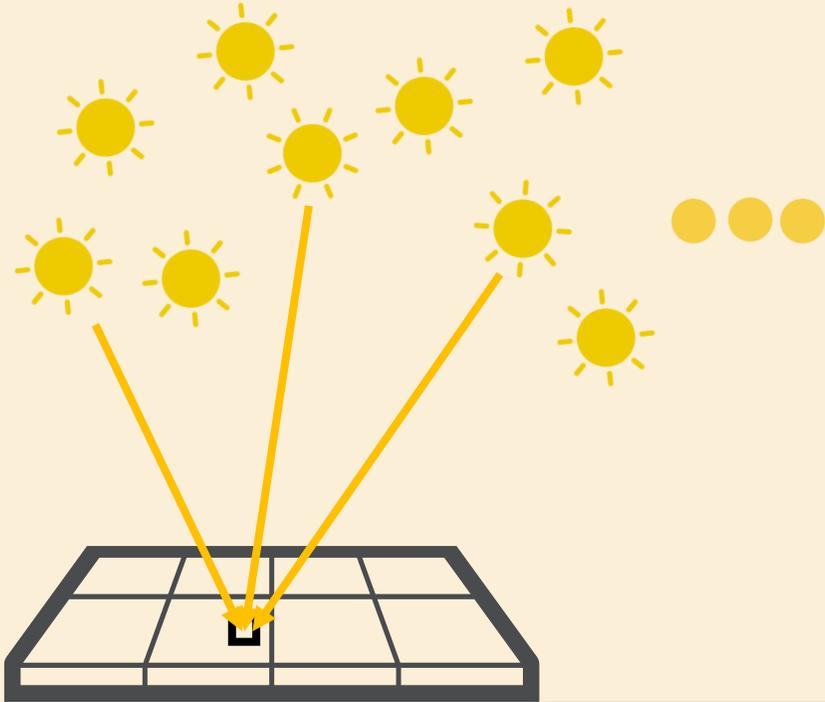
# Multiple Control Variates vs. Single Control Variate



# Multiple Control Variates vs. Single Control Variate

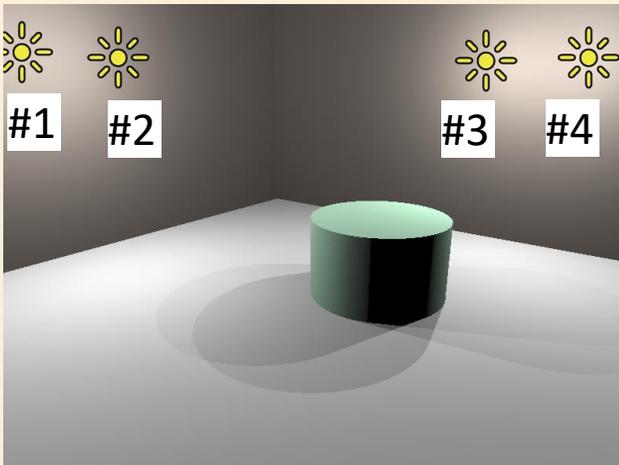


# Results of Multiple Control Variates

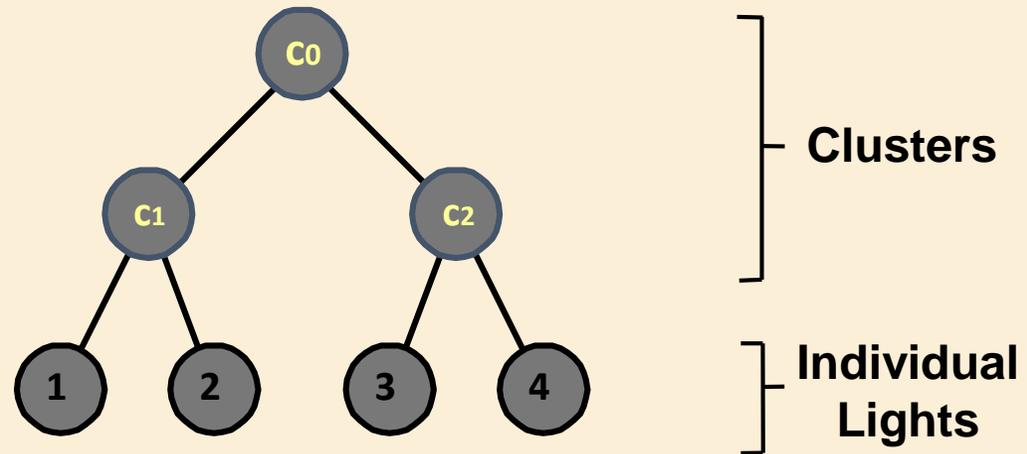


Scalability with Many-Light Scenes!

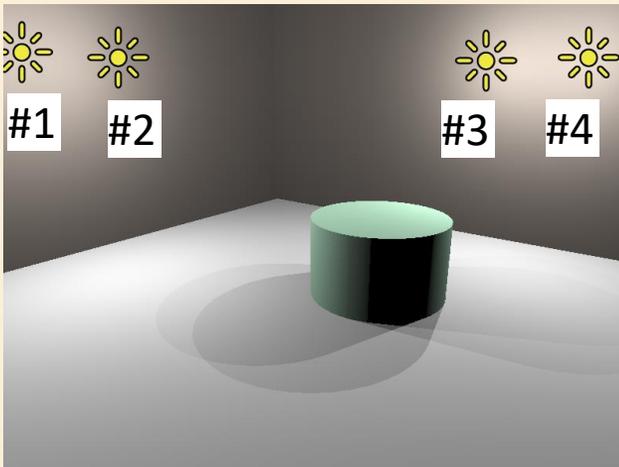
## Simple Example



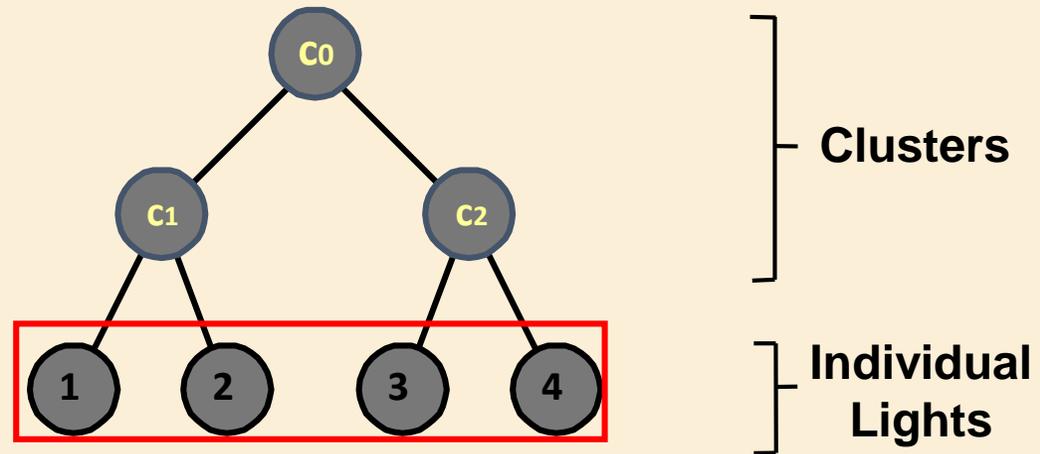
## Light Tree



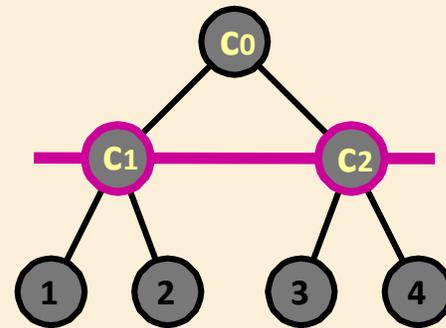
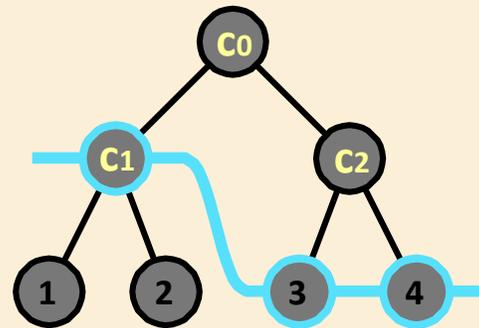
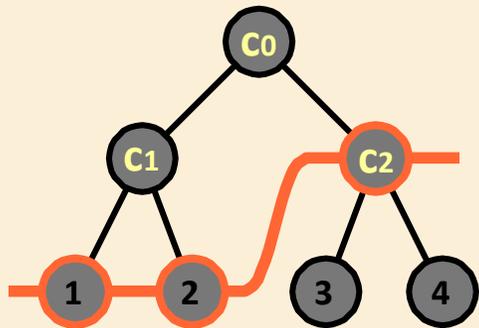
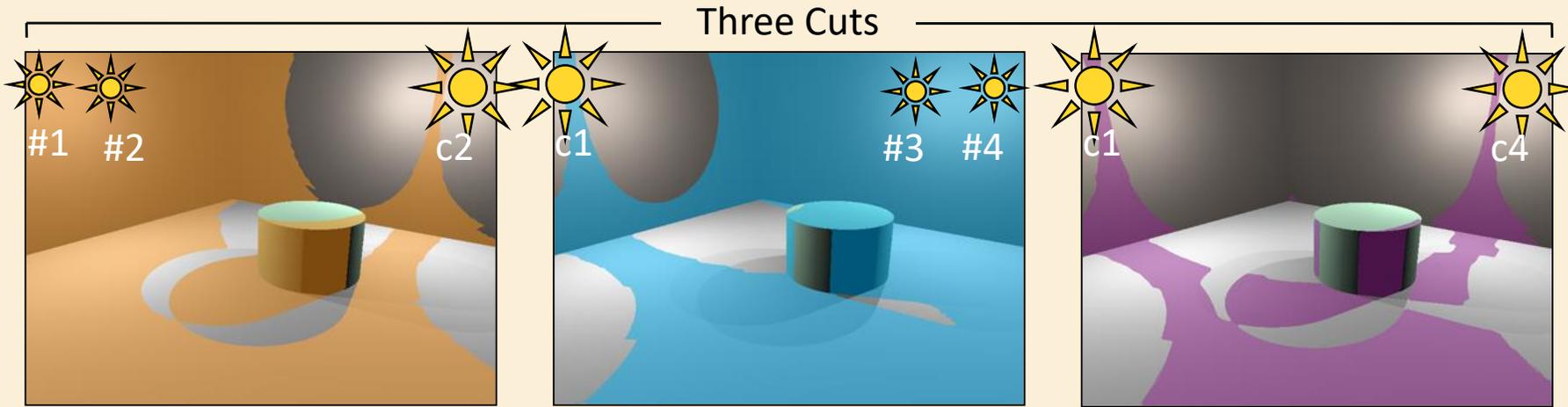
## Simple Example



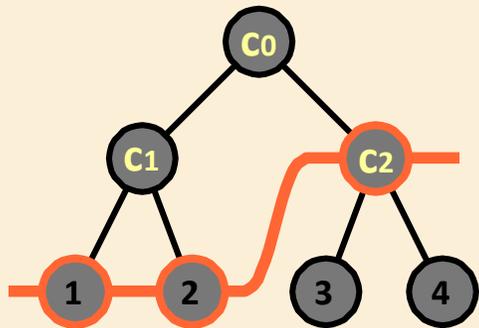
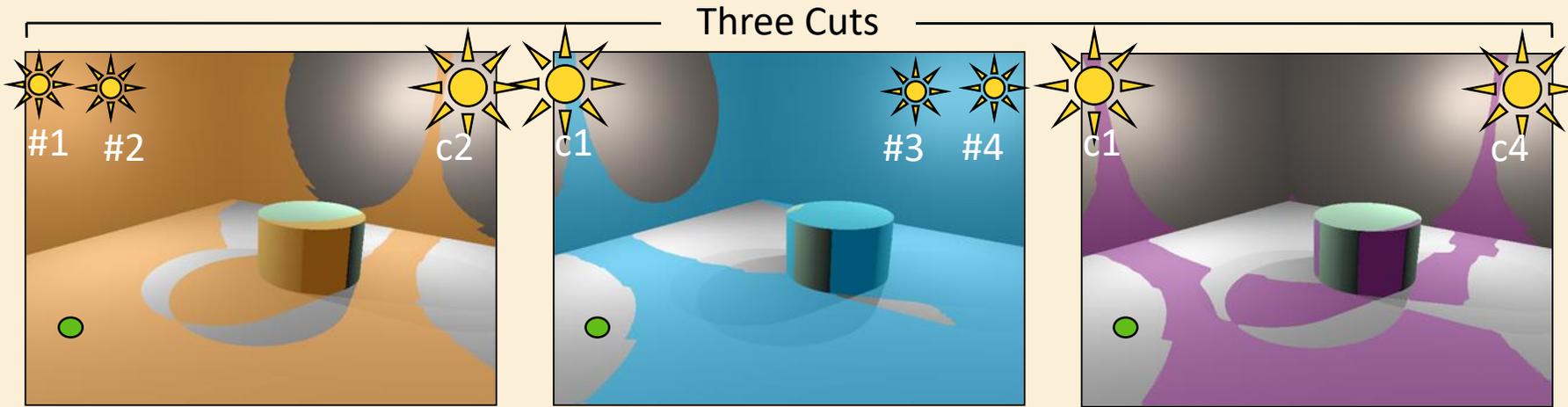
## Light Tree



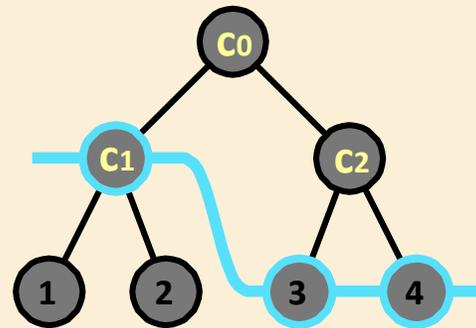
# Lightcuts



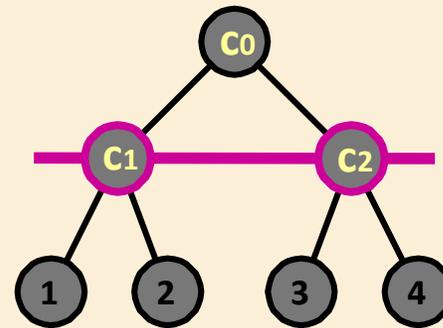
# Lightcuts



**Good**

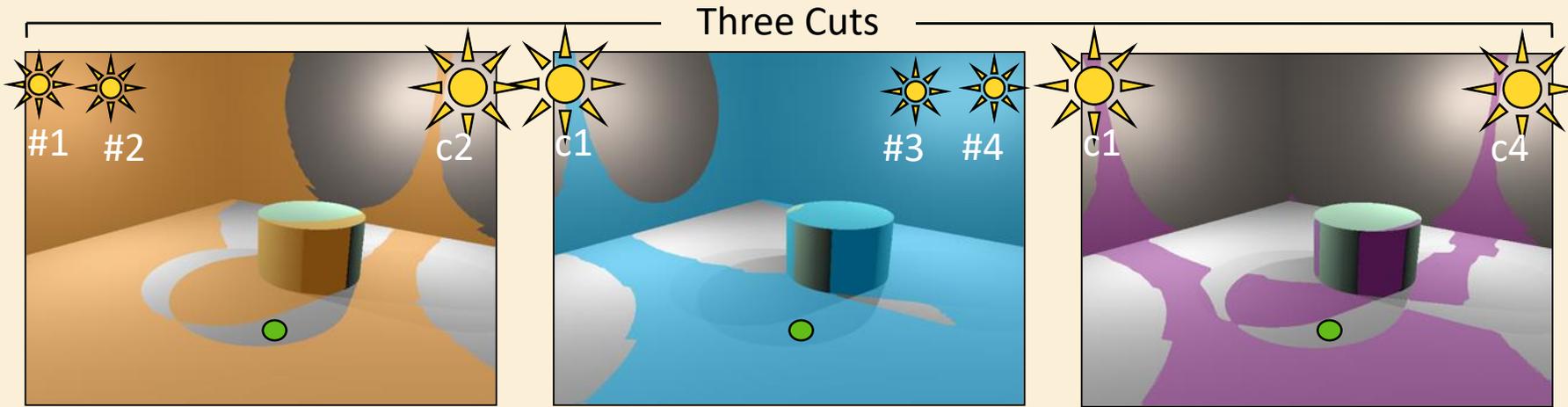


**Bad**



**Bad**

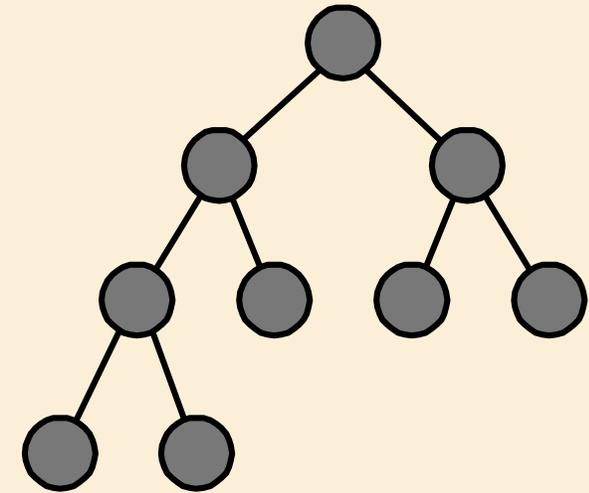
# Lightcuts



Spatially adaptive control variates!

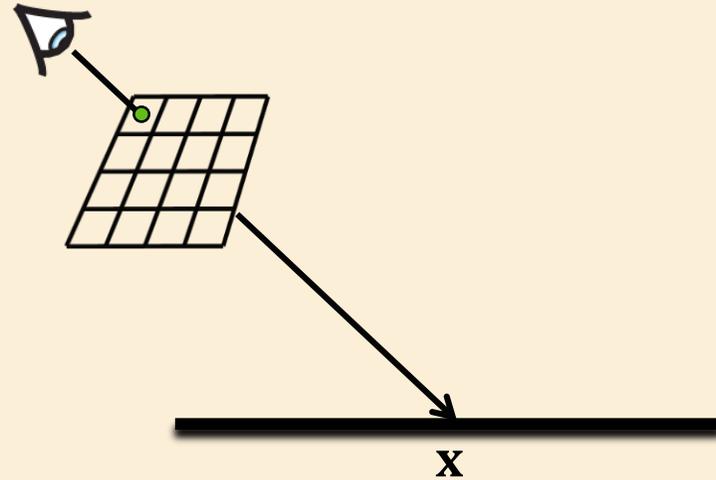
# Multiple Control Variates Construction Based on Lightcuts

- For all lights, we first construct a light BVH tree.



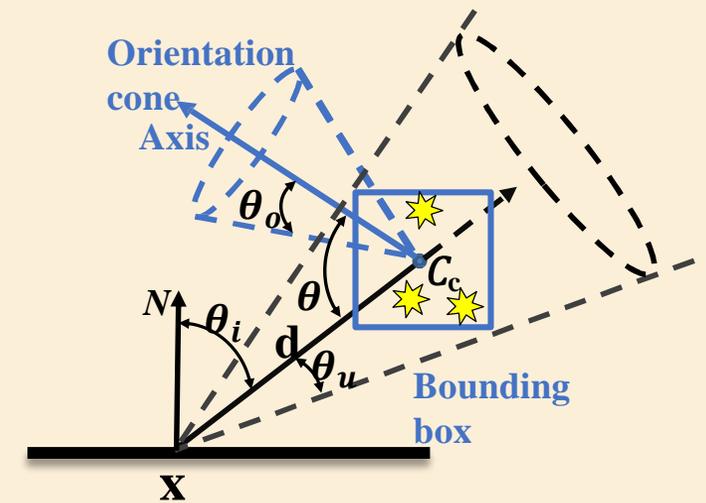
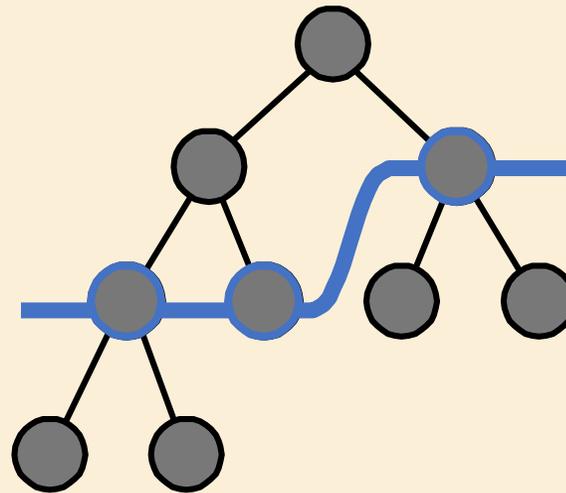
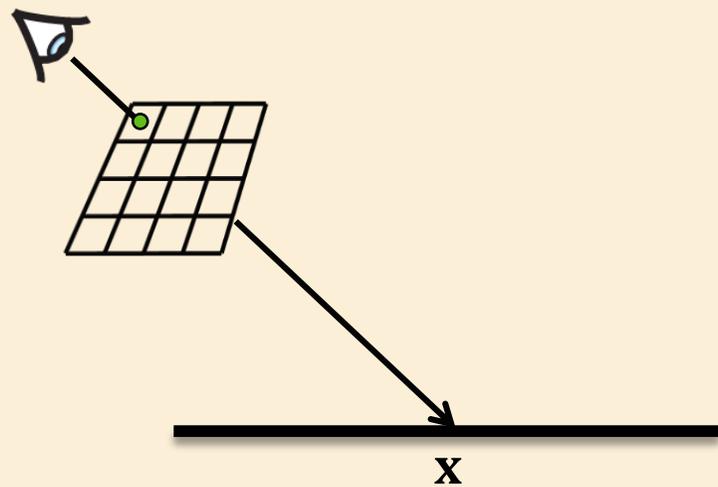
# Multiple Control Variates Construction Based on Lightcuts

- For all lights, we first construct a light BVH tree.
- For each pixel, we select a representative point and shoot a camera ray to obtain the corresponding shading point  $\mathbf{x}$ .



# Multiple Control Variates Construction Based on Lightcuts

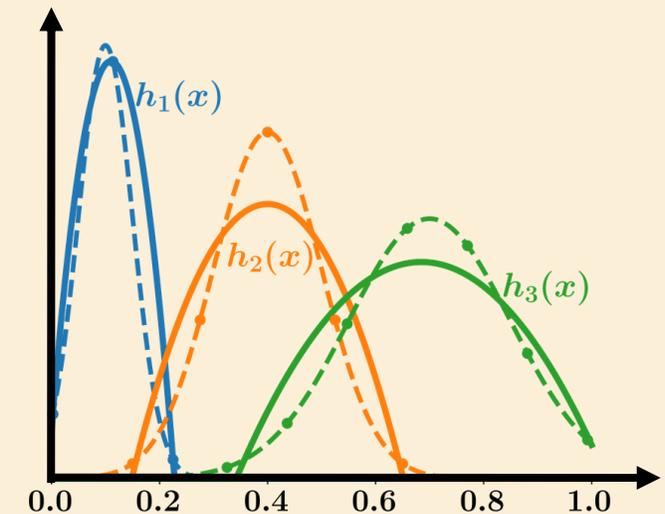
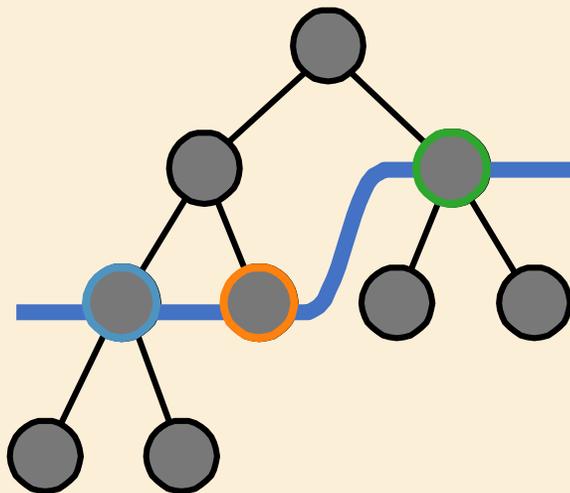
- For all lights, we first construct a light BVH tree.
- For each pixel, we select a representative point and shoot a camera ray to obtain the corresponding shading point  $\mathbf{x}$ .
- For a shading point, we construct a light cut and compute the corresponding weight.



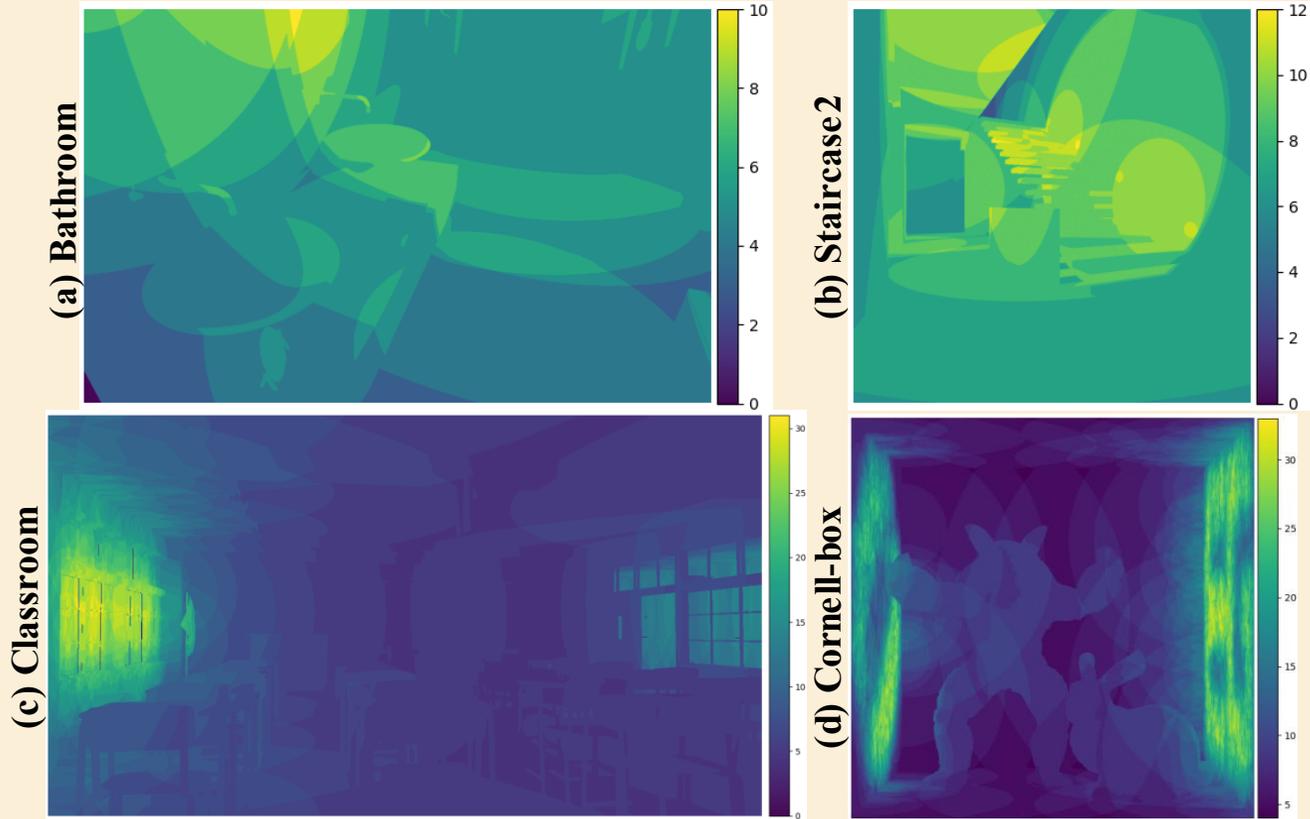
$$I_C(\mathbf{x}) = \frac{f_x |\cos \theta_i| E_c \cos(\theta')}{d^2}$$

# Multiple Control Variates Construction Based on Lightcuts

- For all lights, we first construct a light BVH tree.
- For each pixel, we select a representative point and shoot a camera ray to obtain the corresponding shading point  $\mathbf{x}$ .
- For a shading point, we construct a light cut and compute the corresponding weight.
- For each node in the cut, we create a low-order polynomial as a control variate.



# ■ Visualization of the Number of Control Variates



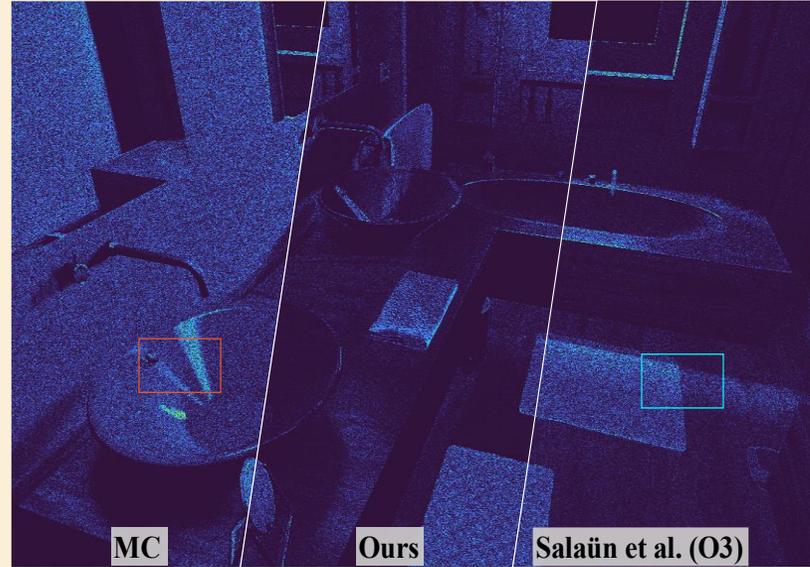
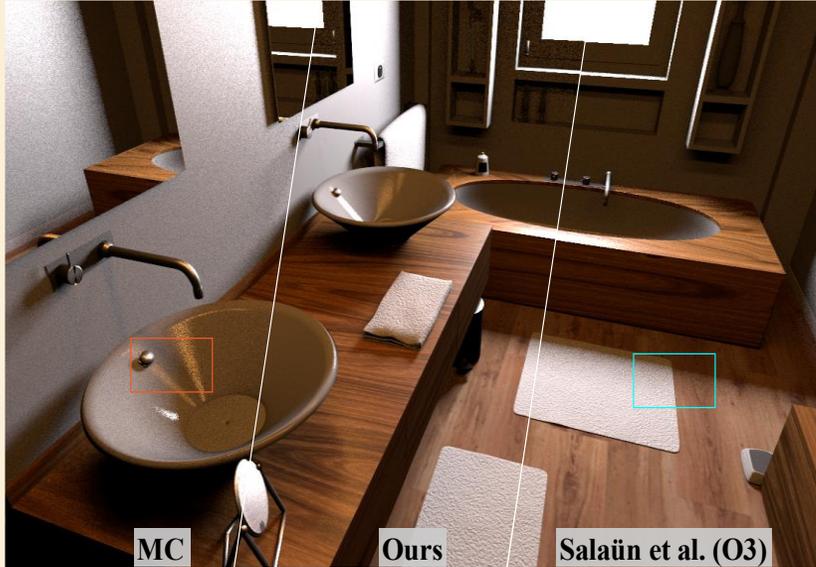
Scene	# lights	avg. cut size	# control variates
Staircase2	21	7.9	0 - 12
Bathroom	32	5.1	0 - 10
Classroom	162	7.6	0 - 31
Cornell-box	1494	10.28	4 - 33

# **Results (Equal Time Comparison)**

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# Comparison to Salaün et al. [SGH\*22]

Bathroom 1200 × 760 pixels



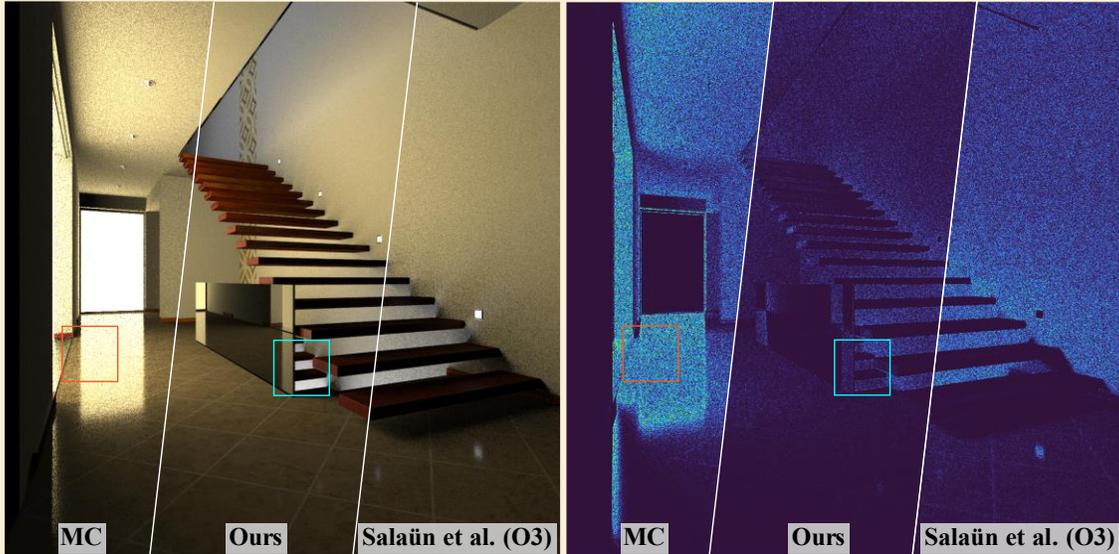
Bathroom scene has 32 mesh lights.

Reference	MC	Ours	Salaün et al. (O1)	Salaün et al. (O3)
				
				
65536 spp relMSE ( $\times 10^{-3}$ )	256 spp 16.8 (1.00 $\times$ )	148 spp 3.6 (4.67 $\times$ )	244 spp 11.9 (1.41 $\times$ ) 	225 spp 9.2 (1.83 $\times$ ) 

**Ours**  
 0-10 (avg. 5.1) control variates  
 (polynomial with order 1)  
**vs.**  
**[SGH\*22]**  
 Single control variate  
 (polynomials with order 1 & 3)

# Comparison to Salaün et al. [SGH\*22]

Staircase2 1024 × 1024 pixels



Staircase2 scene has 21 mesh lights.

**Ours**

0-12 (avg. 7.9) control variates  
(polynomial with order 1)

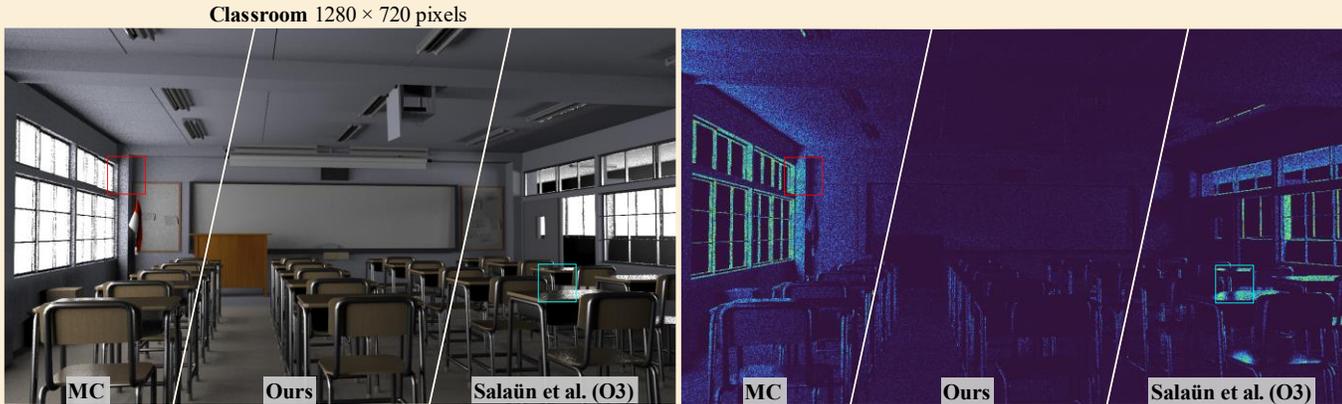
vs.

**[SGH\*22]**

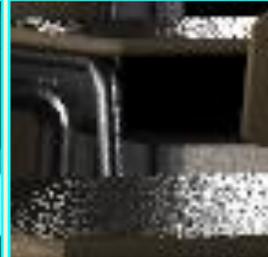
Single control variate  
(polynomials with order 1 & 3)

Reference	MC	Ours	Salaün et al. (O1)	Salaün et al. (O3)
				
				
65536 spp relMSE ( $\times 10^{-3}$ )	256 spp 30.6 (1.00 $\times$ )	112 spp 4.2 (7.28 $\times$ )	224 spp 32.8 (0.93 $\times$ )	206 spp 27.4 (1.11 $\times$ )

# Comparison to Salaün et al. [SGH\*22]

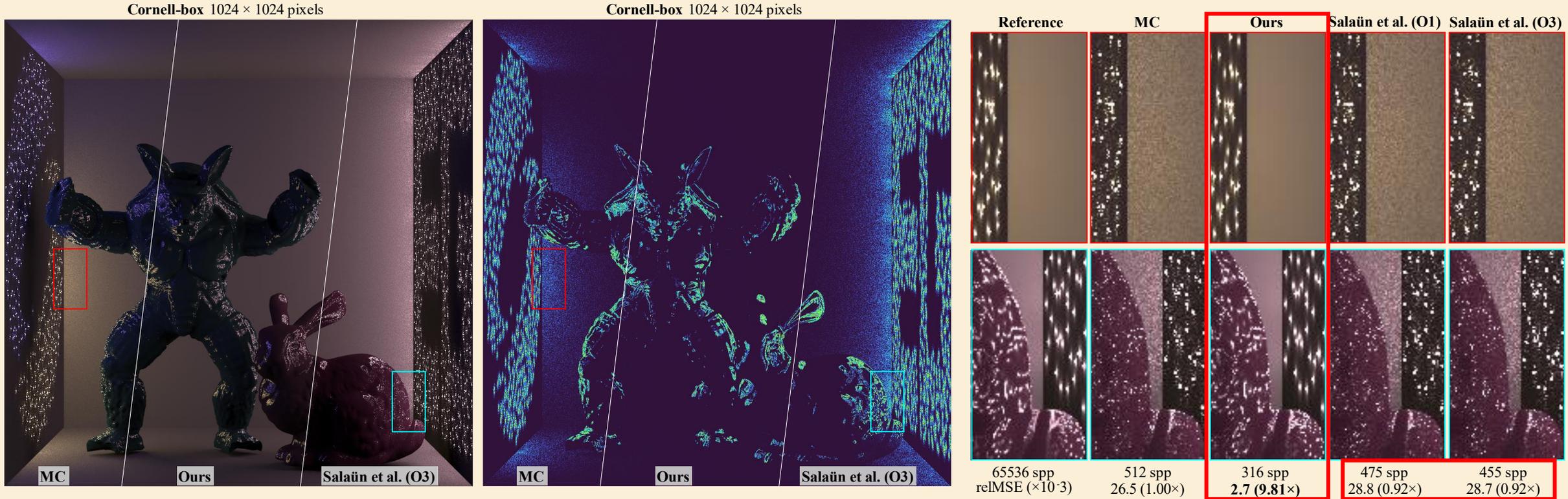


Classroom scene has 162 mesh lights.

Reference	MC	Ours	Salaün et al. (O1)	Salaün et al. (O3)
				
				
65536 spp relMSE ( $\times 10^{-3}$ )	512 spp 20.1 (1.00 $\times$ )	216 spp 2.7 (7.44 $\times$ )	480 spp 28.6 (0.70 $\times$ )	444 spp 22.6 (0.89 $\times$ )

**Ours**  
 0-31 (avg. 7.6) control variates  
 (polynomial with order 1)  
**vs.**  
**[SGH\*22]**  
 Single control variate  
 (polynomials with order 1 & 3)

# Comparison to Salaün et al. [SGH\*22]

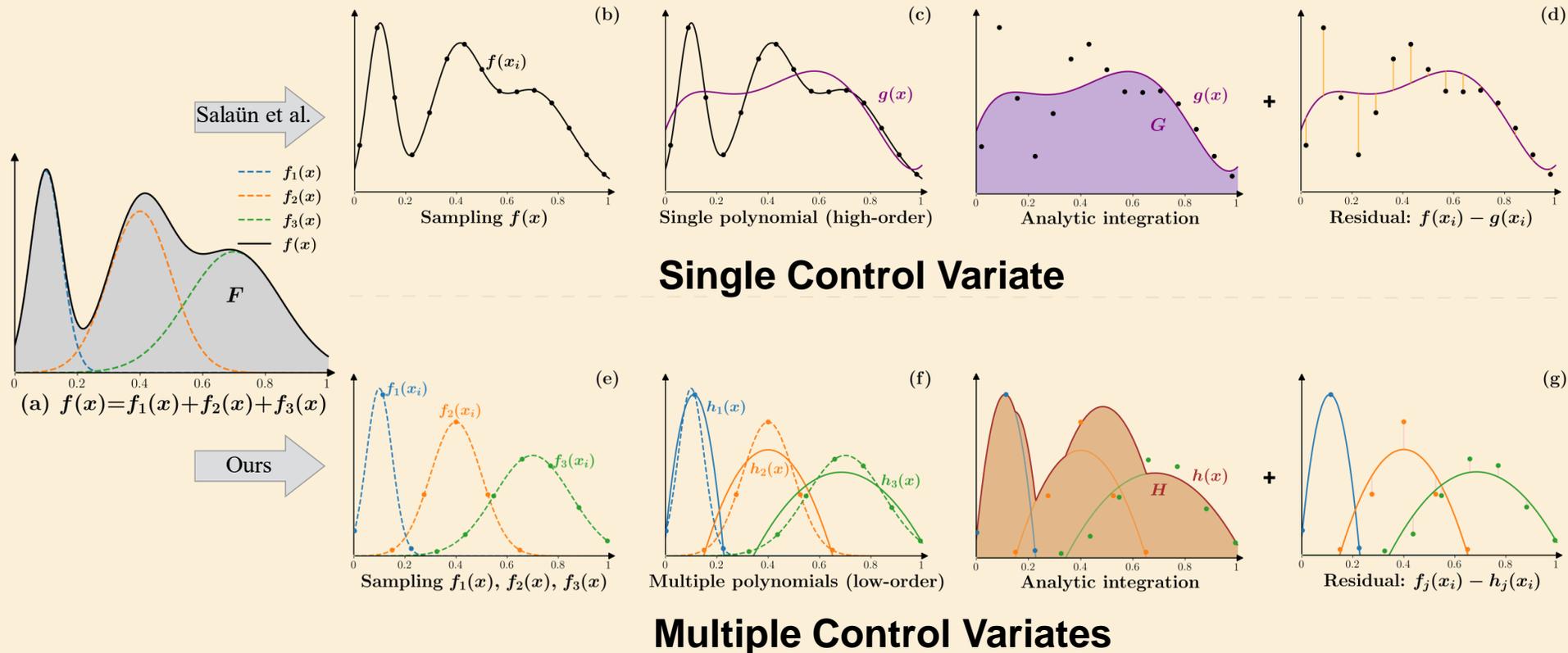


Cornell-box scene has 1494 lights.

**Ours**  
 4-33 (avg. 10.28) control variates (polynomial with order 1)  
**vs.**  
**[SGH\*22]**  
 Single control variate (polynomials with order 1 & 3)

# Conclusion

- We introduce an adaptive multiple control variates framework to improve the Monte Carlo integration for many-light rendering.



# ■ Limitations and Future Work

## Limitations

- Limited to specific many-light rendering.
- Visibility-related discontinuities are not yet handled.

## Future work

- Extend to support other rendering applications.
- Incorporate visibility-aware strategies.

## ■ Acknowledgements

- We would like to thank the reviewers for the valuable comments.
- We would like to thank the following for scenes used in our experiments:  
nacimus (Bathroom), Veach (Veach-mis), NewSee2l035 (Staircase2),  
JayArtist (Living-room2), NovaZeeke (Classroom), Wig42 (Staircase1).
- We would like to thank Corentin Salaün for sharing the code and slides.

# Thank you!

## Adaptive Multiple Control Variates for Many-Light Rendering

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Shandong University, China

